Electromagnetic Responses of 3D Topological Insulators

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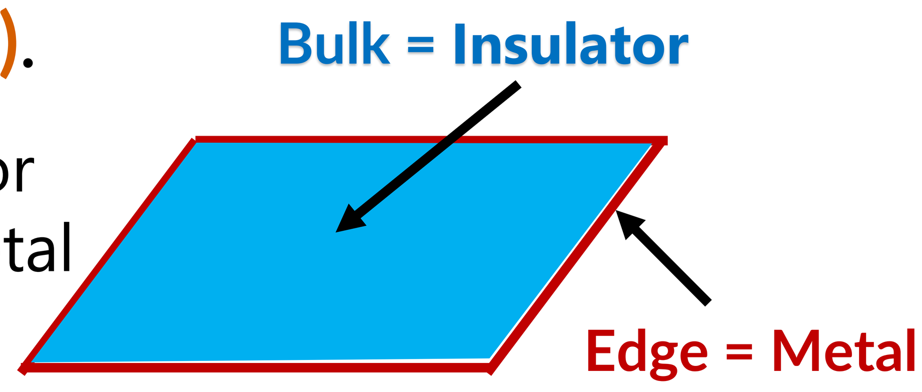
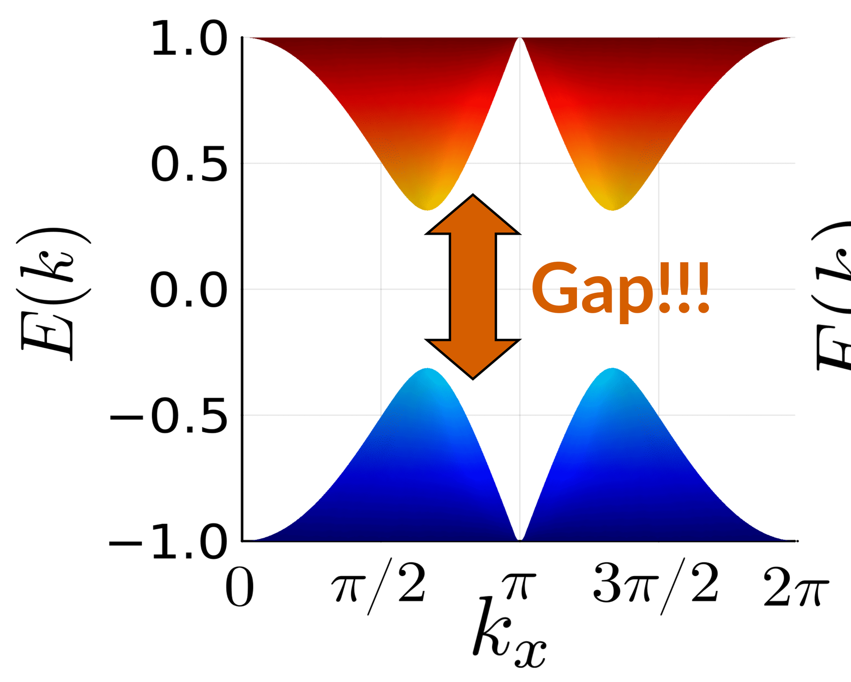
1. What are Topological Insulators?

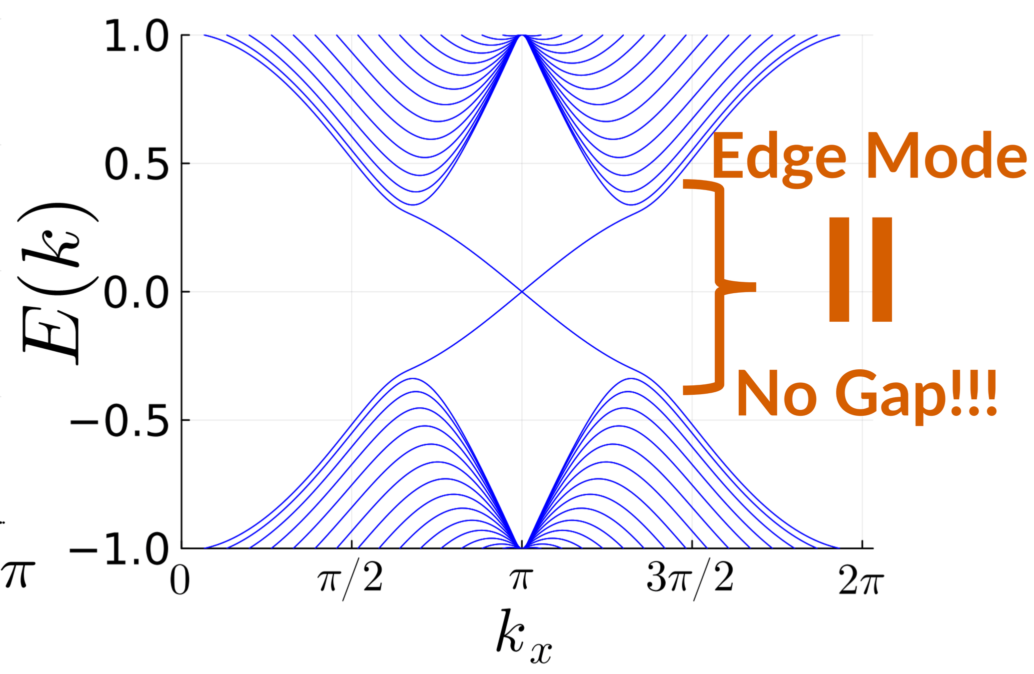
◆In 2005, Kane and Mele proposed **Topological Insulators (TIs)**.

- Bulk → Energy Gap = Insulator
- Edge → No Energy Gap = Metal





Periodic Boundary Condition



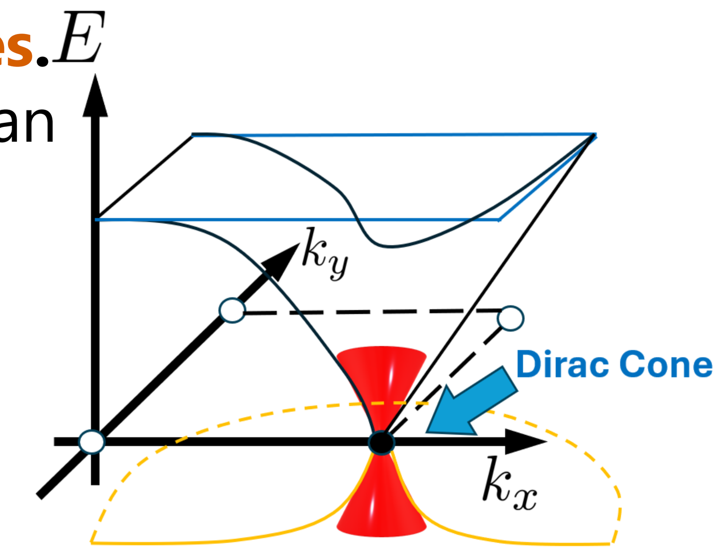
Open Boundary Condition

2. 3D Topological Insulators

◆In 3D, there exist **Strong Topological Insulators**.

- Robust against any perturbations that satisfy **Time Reversal symmetry (TRS)**.
- In the surface Brillouin zone, there is **an odd number of Dirac cones**.
- Strong topological insulators can be distinguished by ν_0,

$$(-1)^{\nu_0} = \prod_{i=1}^{8} \delta(\Gamma_i)$$



Surface Bands

ν_0 is **Topological Invariant**,
$\delta(\Gamma_i)$ is the **parity** at **time-reversal invariant momenta**.

- $\nu_0 = 0$ → **Trivial Insulators**.
- $\nu_0 = 1$ → **Strong Topological Insulators**.

3. Questions

◆In condensed matter physics, insulators and metals are defined by a current response to $\boldsymbol{E}$.

◆Topological Insulators contain both properties.

How do the Maxwell Equations change in Topological Insulators?

4. Topological Field Theory

◆We construct a **Topological Field Theory** that describes **Topological Responses** of 3D Topological Insulators.

$$S_\theta = \frac{e^2}{4\pi^2\hbar c}\int d^3x dt\ \theta(t,\boldsymbol{x})\boldsymbol{E}(t,\boldsymbol{x})\cdot\boldsymbol{B}(t,\boldsymbol{x})$$

- $\theta = 0$ → **Trivial Insulators**.
- $\theta = \pi$ → **Strong Topological Insulators**.

5. Topological Responses in 3D TIs

◆The Modified Maxwell Equations in 3DTIs

$$\nabla\cdot\boldsymbol{D} = 4\pi\rho$$

$$\nabla\times\boldsymbol{H} - \frac{1}{c}\frac{\partial\boldsymbol{D}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}$$

$$\nabla\cdot\boldsymbol{B} = 0$$

$$\nabla\times\boldsymbol{E} + \frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} = 0$$

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P} + \frac{\alpha}{\pi}\theta\boldsymbol{B} = \epsilon\boldsymbol{E} + \frac{\alpha}{\pi}\theta\boldsymbol{B}$$

$$\boldsymbol{H} = \boldsymbol{B} - 4\pi\boldsymbol{M} - \frac{\alpha}{\pi}\theta\boldsymbol{E} = \frac{\boldsymbol{B}}{\mu} - \frac{\alpha}{\pi}\theta\boldsymbol{E}$$

α: Fine structure constant
$\theta = \pi$: Axion Field

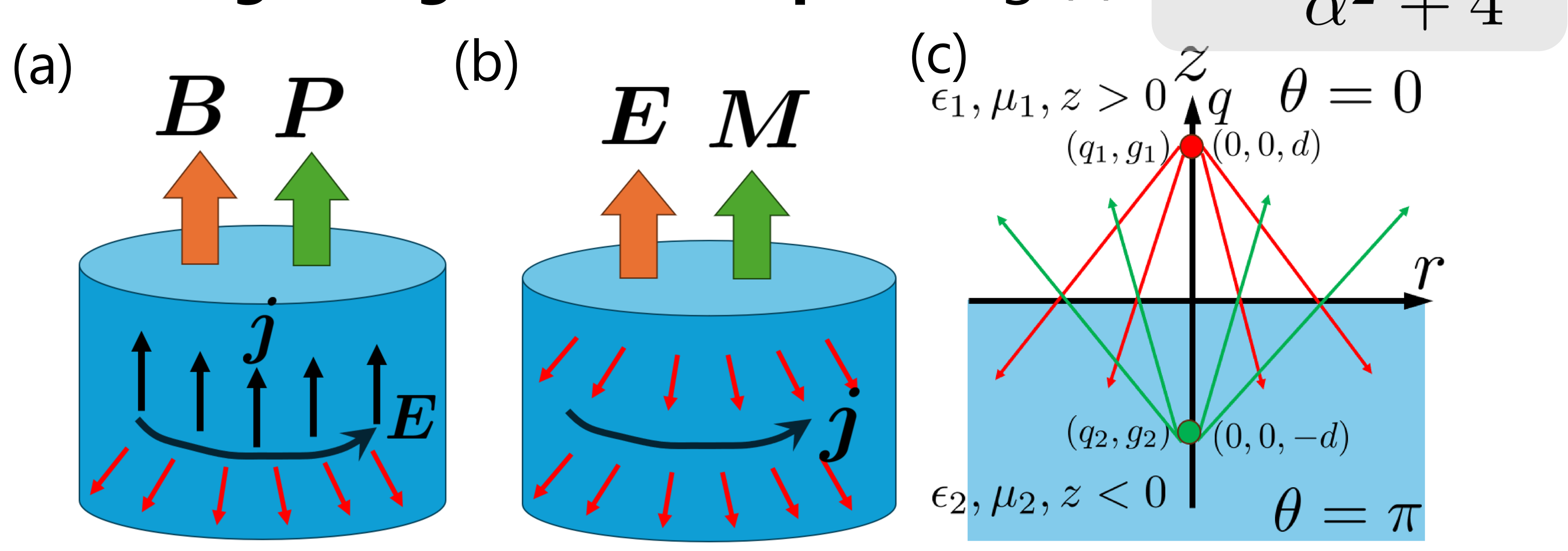
Axion Electrodynamics!

◆**Topological Magnetoelectric Effect** Fig (a), (b)

$$\boldsymbol{P} = \frac{\alpha}{4\pi}\boldsymbol{B},\ \boldsymbol{M} = \frac{\alpha}{4\pi}\boldsymbol{E}$$

◆**Image Magnetic Monopole** Fig (c)

$$g = \frac{2\alpha}{\alpha^2+4}q$$



6. Axion Electrodynamics in Materials

◆ There are some models that realize Axion Electrodynamics in condensed matter.

◆**Antiferromagnetic 3D Topological Insulators**

- 3D TIs + fluctuating antiferromagnetic interaction

$$\theta(t,\boldsymbol{x}) = \frac{\pi}{2}(1+\mathrm{sgn}(m)) - \arctan\frac{n(t,\boldsymbol{x})}{m}$$

$m, n(t,\boldsymbol{x})$ are the gap and the direction of spin receptivity.

◆**Weyl Semimetals**

- 3D Topological Insulators with No energy gap

$$\theta(t,\boldsymbol{x}) = 2(\boldsymbol{b}\cdot\boldsymbol{x} - \mu_5 t)$$

$b_\mu = (\mu_5, -\boldsymbol{b})$ is the chiral gauge field.

7. Conclusion

- We derive nontrivial topological electromagnetic responses of 3D TIs.
- Antiferromagnetic 3D TIs and Weyl Semimetals can also realize Axion Electrodynamics with a dynamical $\theta(t,\boldsymbol{x})$.

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[4] A. A. Zyuzin and A. A. Burkov, Phys. Rev. B **86**, 115133 (2012).